Supporting Information for "State-dependence of polar amplification in an idealized GCM"

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1. Text S1

Here we briefly outline an argument for why the radiative forcing and spatially-varying Planck feedback weakly offset each other in Figure 3 of the main text.

The radiative forcing and spatially-varying Planck feedback have somewhat offsetting contributions to $\widetilde{\mathrm{PA}}$ in Figure 3. This arises from their common dependence on the control climate's emission temperature. As noted in the main text, for a gray radiation scheme such as ours the outgoing longwave radiation perturbation from a unit of warming is $\approx (4\sigma\overline{T}_{em}^3) \times 1\,\mathrm{K}$, with emission temperature T_{em}. The emission temperature is the temperature at the pressure level where the optical depth is unity. The outgoing longwave radiation perturbation from radiative forcing is

$$\approx (4\sigma\overline{T}_{em}^3) \times \delta p_{em} \left(\frac{\partial T}{\partial p_{em}} \right), \tag{1}$$

with emission pressure p_{em} and its changes are negative as the additional longwave absorber shifts the emission to lower pressure. It is clear that the dependence on the third power of the climatological emission temperature cancels between these two radiative flux components. However, these components do not have to perfectly cancel as the extent to which the temperature shift in emission ($\delta p_{em} \partial_{p_{em}} T$) is close to $1\,\mathrm{K}$ can vary across the climate states. In the net, the sum of Fig. 3d and 3e leaves a modest state-dependence that contributes to the larger $\widetilde{\mathrm{PA}}$ in warmer climates with larger equator-to-pole temperature contrasts.

2. Text S2

To derive Equation (8) of the main text, first note that (following Equation (5) of the main text) the surface albedo contribution to warming is

$$\Delta T_{\text{albedo}} = \frac{-\Delta T_s \, \lambda_{\text{albedo}}}{\langle \lambda_P \rangle}. \tag{2}$$

We now derive an estimate for λ_{albedo} at the ice edge. To do this we first need expressions for the insolation and the sensitivity of albedo to temperature, both at $SST = T_{\text{edge}}$. An expression for the insolation as a function of the control climate SST distribution, can be obtained by combining Equations (1) and (3) of the main text,

$$I(SST) = \frac{S_0}{4}\left[1 + \frac{3\Delta_s}{4\Delta_h}\left(T_0 - SST\right)\right], \tag{3}$$

and a first-order Taylor expansion of Equation (2)) around $SST = T_{\text{edge}}$ also provides the sensitivity of surface albedo to temperature changes near the ice-edge,

$$\left.\frac{d\alpha}{dT}\right|_{\text{edge}} = \frac{\alpha_0 - \alpha_i}{2h}. \tag{4}$$

Now we imagine that, in the absence of ice-albedo feedbacks, the temperature change in response to forcing is a globally-uniform value $= \langle \Delta T_s \rangle$. This uniform warming will cause a change in the albedo at the ice edge of $\frac{d\alpha}{dT} \times \langle \Delta T_s \rangle$. Recalling that our GCM's atmosphere is transparent in the shortwave, and assuming that changes in the top-of-atmosphere insolation at the ice-edge induced by shifts in the ice-edge latitude are small, we can estimate the change in downward solar flux at the surface (i.e., the absorbed solar flux) as,

$$\Delta \text{SW}^{\downarrow}_{\text{sfc}} \approx I(T_{\text{edge}}) \times \frac{d\alpha}{dT} \times \langle \Delta T_s \rangle\,, \tag{5}$$

where $I(T_{\mathrm{edge}})$ is Eq. 3 (above) evaluated at $SST = T_{\mathrm{edge}}$. This yields a simple estimate for $\lambda_{\mathrm{albedo}}$ at the ice edge,

$$\lambda_{\mathrm{albedo}} \approx I(T_{\mathrm{edge}}) \times \left.\frac{d\alpha}{dT}\right|_{\mathrm{edge}}. \tag{6}$$

Plugging this into Equation (1) (above), and dividing by the global-mean temperature change to get $dT_{\mathrm{albedo}}/d\left\langle T_s \right\rangle$, yields Equation (8) of the main text.

References

Chang, C.-Y., & Merlis, T. M. (2023). The role of diffusivity changes on the pattern of warming in energy balance models. *J. Climate*, *36*, 7993-8006.

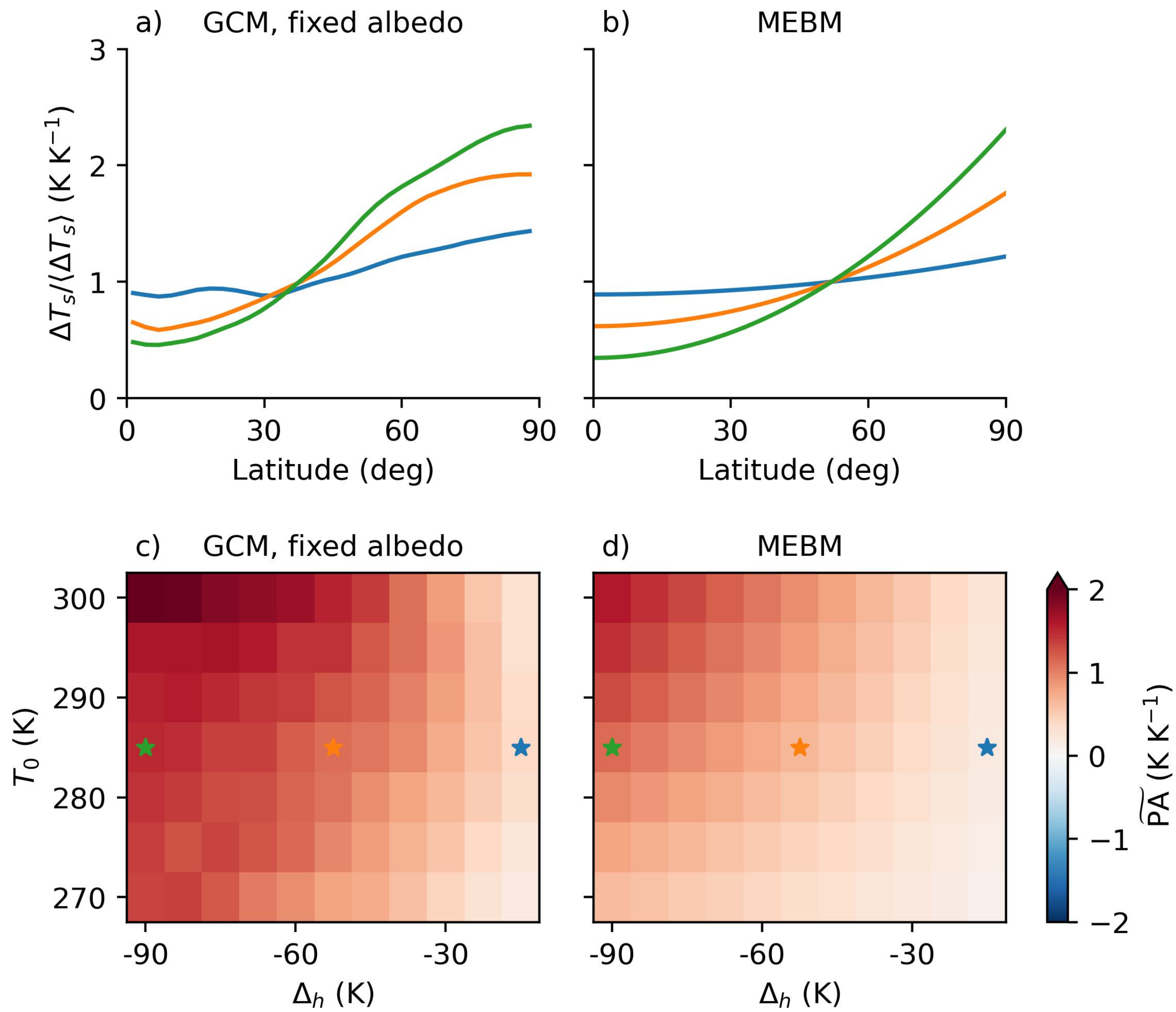

Figure S1. As in Figure 1 of the main text, but comparing the GCM simulations with fixed surface albedo to the analytical MEBM model of Chang and Merlis (2023) (their Equation 7). The local surface temperature changes in panels (a) and (b) are normalized by the global-mean warming, and are the same illustrative examples as in Figure 1 of the main text.

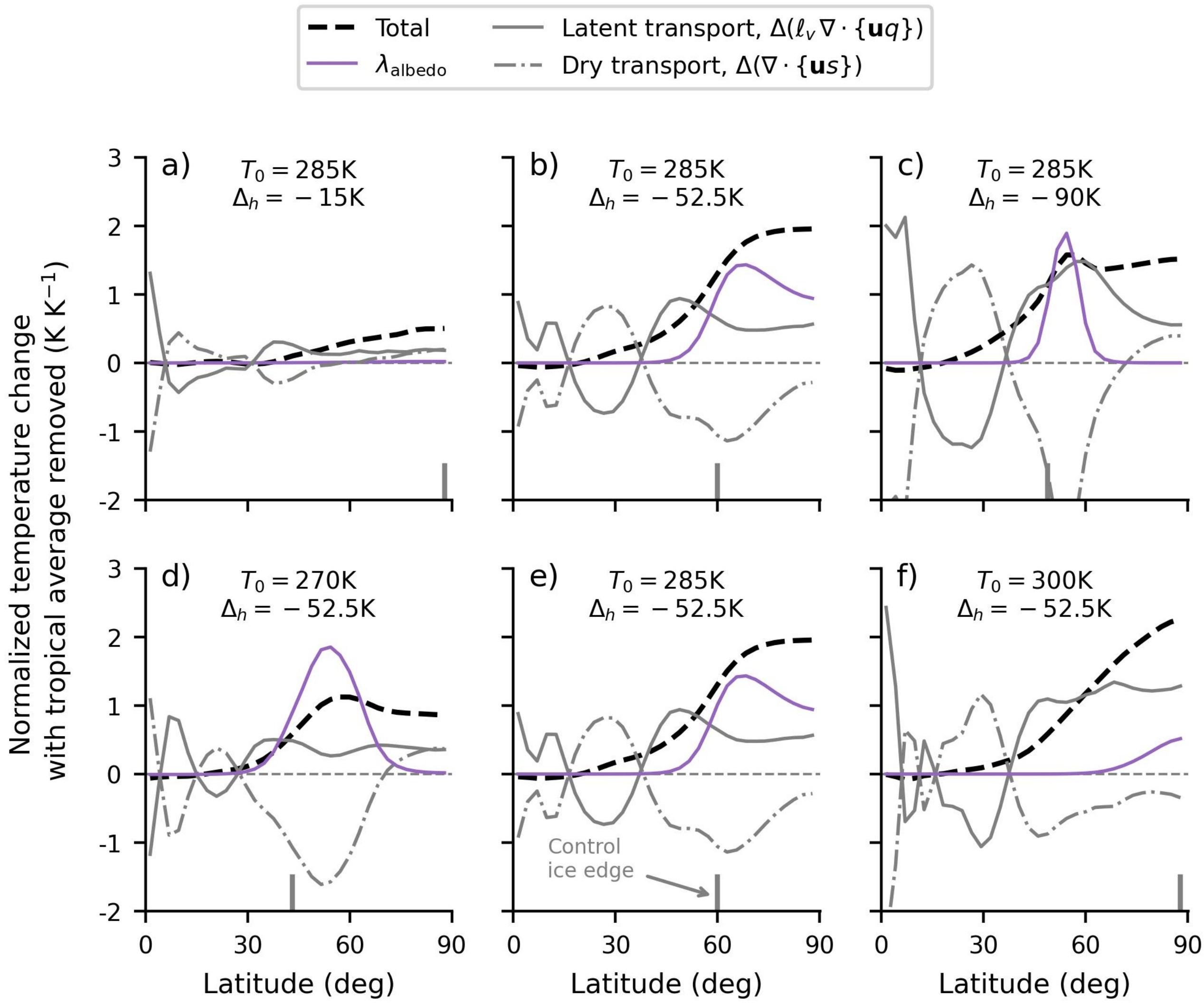

Figure S2. As in Figure 2 of the main text, but now showing the decomposition of the divergence of atmospheric energy transport into its dry and latent components. The takeaways from this figure are: firstly, that the local temperature perturbation from the surface albedo feedback is balanced by divergence of dry static energy, and secondly, that the latent energy transport contribution to polar temperature change increases markedly with increasing T_0 and Δ_h.

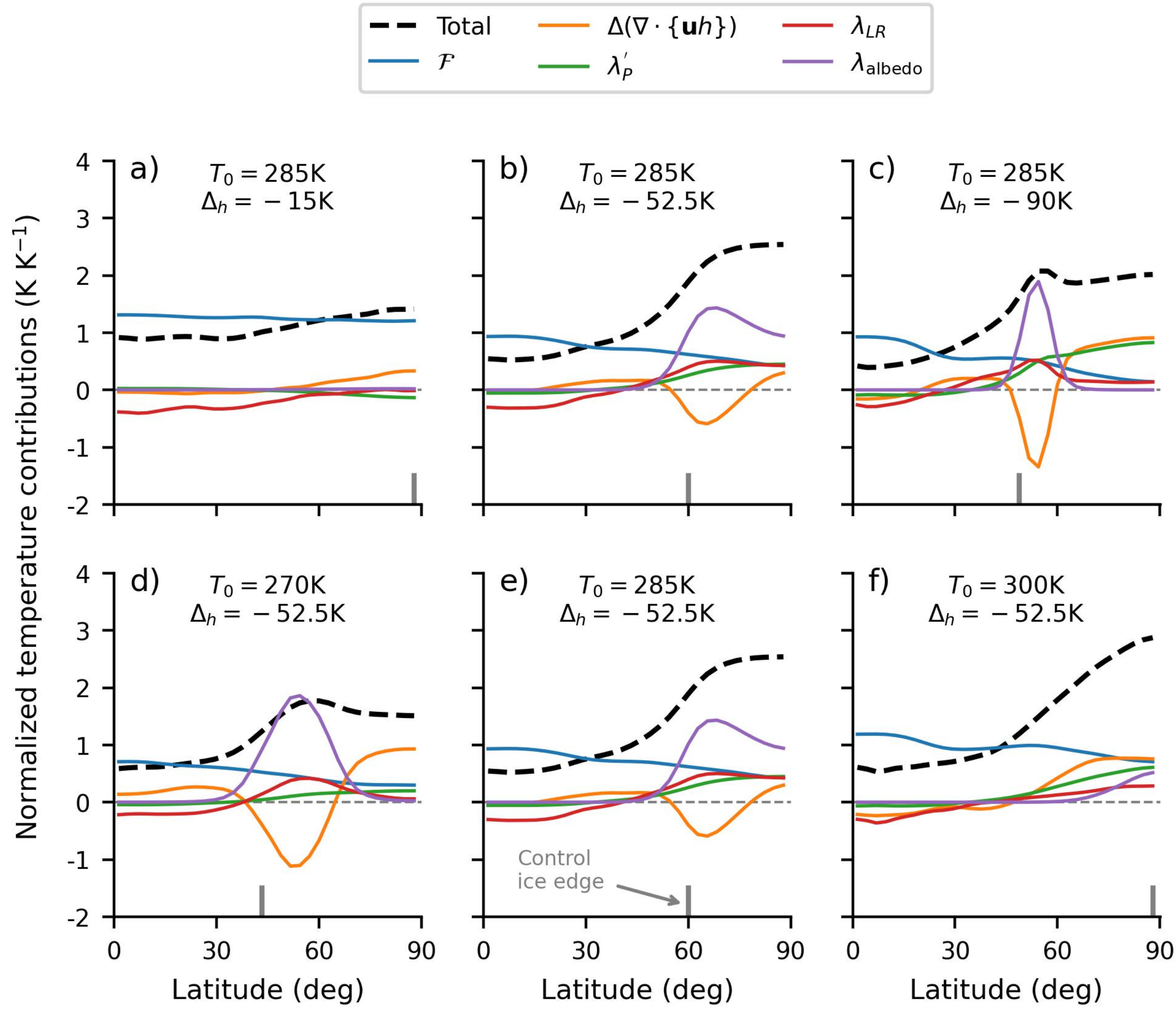

Figure S3. As in Figure 2 of the main text, but without removing the tropical average of each temperature contribution. This figure is useful for seeing the negative tropical lapse-rate contribution to $\widetilde{\mathrm{PA}}$ and the positive local lapse-rate contribution in the vicinity of the surface albedo feedback.

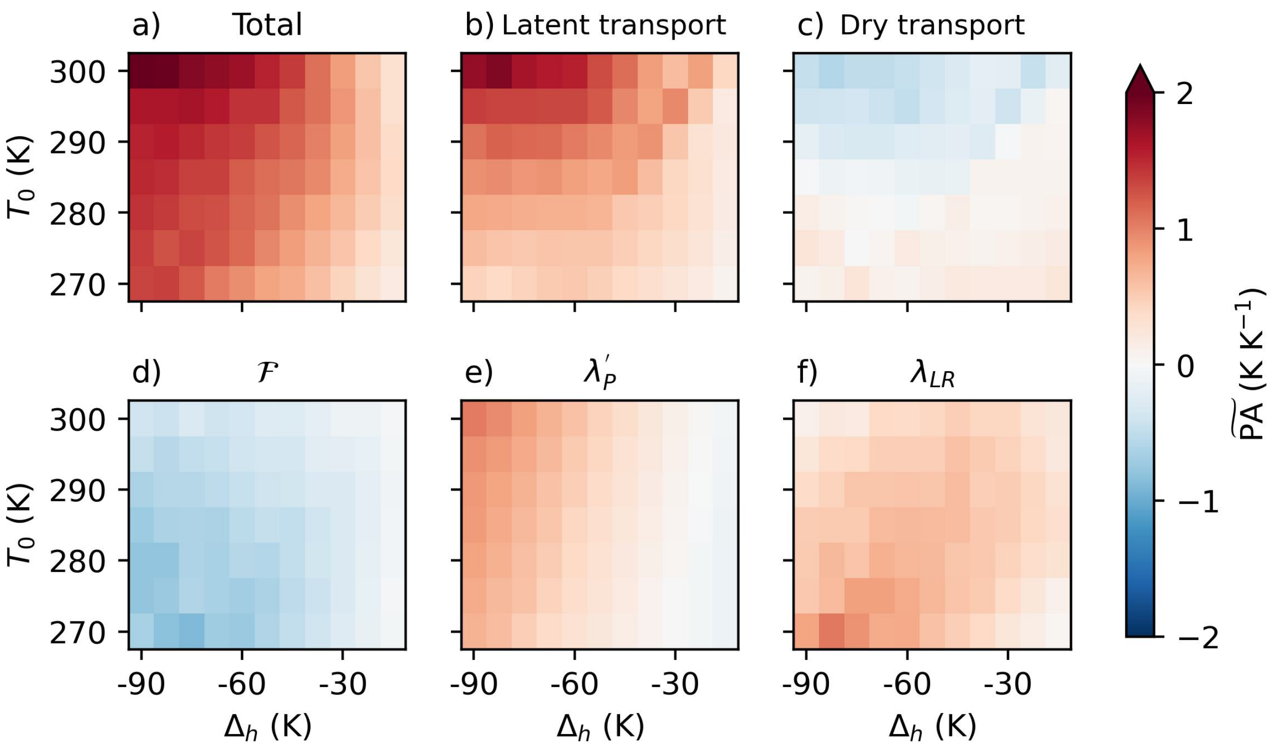

Figure S4. As in Figure 3 of the main text, but for simulations with a fixed, uniform surface albedo. The contributions to $\widetilde{\mathrm{PA}}$ from latent transport, radiative forcing, the spatially-varying Planck feedback, and the lapse-rate feedback, are similar to the simulations in the main text which include an ice-albedo feedback.

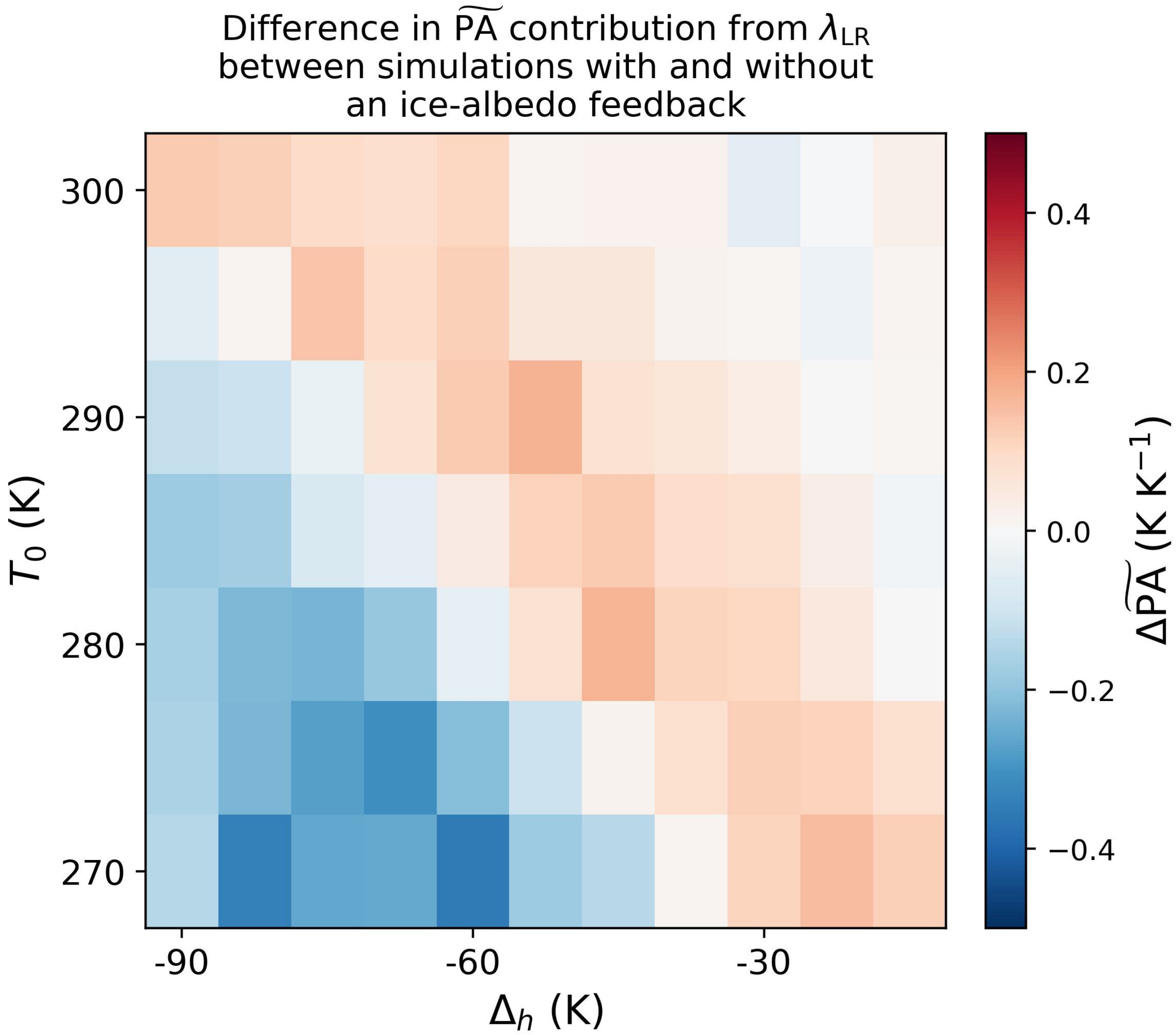

Figure S5. The difference between Figure 2f and Figure S3f. The region in red covers a similar portion of the phase space to the ice-albedo contribution to $\widetilde{\mathrm{PA}}$ in Figure 2a, indicating that the temperature change from the lapse-rate feedback is locally enhanced by the surface albedo feedback.